CHAPTER 6
BOOLEAN ALGEBRA

Q1. What is Boolean algebra?

Ans: In 1847, a British Mathematician, George Boole, introduced Algebra of Binary Numbers, known as Boolean algebra. Boolean algebra is used in designing of logic circuits, used in computer. These circuits perform different logical operations. Boolean algebra is also known as logical algebra or switching algebra.

Q6. Explain the following Logic gates and show their function by using a Truth Table. AND,OR, NOT

Ans: **<u>AND Gate:</u>** AND operation is represented by a dot. It is used for logical multiplication.

X	Y	X.Y
0	0	0
0	1	0
1	0	0
1	1	1

<u>OR Gate:</u> OR operation is represented by a plus. It is used for logical addition.

X	Y	X+Y
0	0	0
0	1	1
1	0	1
1	1	1

<u>Not Gate:</u> The NOT Gate is an inverter gate with only one input and one output signal.

X	$\overline{X}$
0	1
1	0

Q3. Find the values of the Boolean expressions

Ans: **i)** $XY + X\overline{Y}$ when X=1 and Y=0

= 1.0+1.1

= 0 + 1

= 1

ii) (X+Y) . (XY) when X=1 and Y=0

= (1+0) . (1.0)

= 1 . 0

= 0

iii) $(X+\overline{Y}).(\overline{X}+Y)$ when X=1 and Y=1

= (1+0) . (0+1)

= 1 . 1

= 1

Q4. Find out the dual of the following Boolean expressions:

i). $XY + X\overline{Y}$

Solution: $(X+Y).(X+\overline{Y})$

ii). $(X+Y)+(X.Y)$

Solution: $(X.Y).(X+Y)$

iii) $(X+\overline{Y})+(\overline{X}+Y)$

Solution: $(X.\overline{Y}).(\overline{X}.Y)$

Q5. State and prove the two basic Demorgan's theorem.

Ans: Demorgan's Law

It states that the complement of any expression is equal to the complement of each Boolean variable independently if the operators are changed i.e '+' changes to '.' and '.' changes to '+'.

a) $\overline{A+B} = \overline{A}\,.\,\overline{B}$

Proof: $LHS = \overline{A+B}$

$= \overline{A} \;\overline{+}\; \overline{B}$

$= \overline{A} \;.\; \overline{B} \qquad = RHS$

b) $\overline{A.B} = \overline{A} + \overline{B}$

Proof: $\text{LHS} = \overline{A.B}$

$= \overline{A} \;\overline{.}\; \overline{B}$

$= \overline{A} + \overline{B} \qquad = RHS$

Q6. What is a truth table?

Ans: A truth table is a table that shows the result of a Boolean expression for all the possible combinations of the values. For example the truth table for OR operation is as follows:

OR OPERATION		
A	**B**	**X=A . B**
0	0	0
0	1	1
1	0	1
1	1	1

Q7. Construct a truth table for AND and Not of AND operation for the three variables X,Y and Z.

X	Y	Z	X.Y.Z	NOT(X.Y.Z)
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	0

Q8. State the following laws:

Ans: Idempotent Law

It states that sum or product of a variable with itself is equal to that variable i.e. A+A=A and A . A=A.

Absorption Law

(i) The sum of a Boolean variable with the product of another variable & itself is equal to the first variable. i.e. A + (A . B) = A.

(ii) The product of a Boolean variable with the sum of another variable & itself is equal to the first variable. i.e. A . (A + B) = A

Involution Law

It states that double complementation has cancellation effect. i.e. A=A

Q9 Construct a truth table for the following Boolean expression.

Ans. i) $XY+\overline{X}Z+YZ$

X	Y	Z	$\overline{X}$	XY	$\overline{X}Z$	YZ	$XY+\overline{X}Z+YZ$
0	0	0	1	0	0	0	1
0	0	1	1	0	1	0	1
0	1	0	1	0	0	0	0
0	1	1	1	0	1	1	1
1	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0
1	1	0	0	1	0	0	1
1	1	1	0	1	0	1	1

ii). $(X + Y) . (XY)$

X	Y	$X+Y$	XY	$(X+Y).(XY)$
0	0	0	0	0
0	1	1	0	0
1	0	1	0	0
1	1	1	1	1

iii) $X\overline{Y} + XZ + Y\overline{Z}$

X	Y	Z	$\overline{Y}$	$\overline{Z}$	$X\overline{Y}$	XZ	$Y\overline{Z}$	$X\overline{Y} + XZ + Y\overline{Z}$
0	0	0	1	1	0	0	0	0
0	0	1	1	0	0	0	0	0
0	1	0	0	1	0	0	1	1
0	1	1	0	0	0	0	0	0
1	0	0	1	1	1	0	0	1
1	0	1	1	0	1	1	0	1
1	1	0	0	1	0	0	1	1
1	1	1	0	0	0	1	0	1

Q11. Simplify the following Boolean expression.

i). $\overline{A}C + \overline{A}B + A\overline{B}C + BC$	**i).** $\overline{A}C + \overline{A}B + A\overline{B}C + BC$
$= \overline{A}C(B+\overline{B}) + \overline{A}B + A\overline{B}C + BC$	$= \overline{A}C + \overline{A}B + BC + A\overline{B}C$
$= \overline{A}BC + \overline{A}\,\overline{B}C + \overline{A}B + A\overline{B}C + BC$	$= \overline{A}C + \overline{A}B + C(B + A\overline{B})$
$= BC + \overline{A}BC + \overline{A}\,\overline{B}C + A\overline{B}C + \overline{A}B$	$= \overline{A}C + \overline{A}B + C(B+A).(B+\overline{B})$
$= BC(1+\overline{A}) + \overline{B}C(A+\overline{A}) + \overline{A}B$	$= \overline{A}C + \overline{A}B + C.(B+A).1$
$= BC.1 + \overline{B}C.1 + \overline{A}B$	$= \overline{A}C + \overline{A}B + C.(B+A)$
$= BC + \overline{B}C + \overline{A}B$	$= \overline{A}C + AC + \overline{A}B + BC$
$= C(B+\overline{B}) + \overline{A}B$	$= C(A+\overline{A}) + \overline{A}B + BC$
$= C.1 + \overline{A}B$	$= C.1 + \overline{A}B + BC$
$= C + \overline{A}B$	$= C + B.C + \overline{A}B$
OR	$= C(1+B) + \overline{A}B$
	$= C.1 + \overline{A}B = C + \overline{A}B$

ii) $(A+B+C).(A+\overline{B}+\overline{C}).(A+B+\overline{C}).(A+\overline{B}+C)$	**iii).** $X.\overline{Y}.Z + X.\overline{Y}.\overline{Z} + \overline{X}.\overline{Y}.Z + \overline{X}.\overline{Y}.\overline{Z}$
LET $A+B=D$, $A+\overline{B}=E$	$= X.\overline{Y}(Z+\overline{Z}) + \overline{X}.\overline{Y}(Z+\overline{Z})$
$= (D+C).(E+\overline{C}).(D+\overline{C}).(E+C)$	$= X.\overline{Y}.1 + \overline{X}.\overline{Y}.1$
$= (D+C).(D+\overline{C}).(E+C).(E+\overline{C})$	$= X.\overline{Y} + \overline{X}.\overline{Y}$
$= [D+(C.\overline{C})].[E+(C.\overline{C})]$	$= \overline{Y}.(X+\overline{X})$
$= (D+0).(E+0)$	$= \overline{Y}.1$
$= D.E$	$= \overline{Y}$
$= (A+B).(A+\overline{B})$	
$= A+(B.\overline{B})$	
$= A+0$	
$= A$	

Q12. Simplify the following function using karnaugh map.

i) $A\overline{B}C + ABC + \overline{B}\,\overline{C}$

$= A\overline{B}C + ABC + \overline{B}\,\overline{C}(A+\overline{A})$

$= A\overline{B}C + ABC + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C}$

	$\overline{B}\,\overline{C}$	$\overline{B}C$	BC	$B\overline{C}$
$\overline{A}$	**1**			
A	**1**	**1**	**1**	

$= \overline{B}\,\overline{C} + AC$

ii) $ABC+A\overline{B}C+AB\overline{C}$

	$\overline{B}\,\overline{C}$	$\overline{B}C$	BC	$B\overline{C}$
$\overline{A}$				
A		**1**	**1**	**1**

$=AC+AB$

$=A(C+B)$

iii) $A\overline{B}C+\overline{A}B\overline{C}+\overline{A}\,\overline{B}C$

	$\overline{B}\,\overline{C}$	$\overline{B}C$	BC	$B\overline{C}$
$\overline{A}$		**1**		**1**
A		**1**		

$=\overline{B}C+\overline{A}B\overline{C}$

iv) $A\overline{B}+\overline{A}C+B\overline{C}$

$=A\overline{B}(C+\overline{C})+\overline{A}C(B+\overline{B})+B\overline{C}(A+\overline{A})$

$=A\overline{B}C+A\overline{B}\,\overline{C}+\overline{A}BC+\overline{A}\,\overline{B}C+AB\overline{C}+\overline{A}B\overline{C}$

	$\overline{B}\,\overline{C}$	$\overline{B}C$	BC	$B\overline{C}$
$\overline{A}$		**1**	**1**	**1**
A	**1**	**1**		**1**

$=A\overline{B}+\overline{A}C+B\overline{C}$

GENERAL QUESTIONS

Q1. What is Karnaugh map?

Ans: Karnaugh map are used to simplify the Boolean expression. It is used to minimize the number of literal and hence requires less number of gates for its implementation.

Q2. What is Boolean expression?

Ans: In Boolean expression, statements are represented by variables connected or operated by logical operators.

Q3. Prove with the help of truth table that

i). $\overline{A+B}=\overline{A}\,.\,\overline{B}$

A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\,.\,\overline{B}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

ii) $\overline{A.B}=\overline{A}+\overline{B}$

A	B	$A\,.\,B$	$\overline{A.B}$	$\overline{A}$	$\overline{B}$	$\overline{A}+\overline{B}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

Q4: Define Logic Gate.

Ans: A Gate is a circuit that controls the flow of information performing logical operations.

Q5. Define the AND, OR and NOT Gates.

AND Gate: The AND Gate has two or more input signals and one output signal. It is used for AND operation.

OR Gate: The OR Gate has two or more input signals and one output signal. It is used for OR operation.

NOT Gate: The NOT Gate is an inverter gate with only one input signal and one output signal.